Reasoning with Partial Orders

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SUMMARY.
The present study is concerned with the Ignorance Inferences that come with Superlative Modifiers (SMs) like *at least* and *at most*. Despite all the increasing interesting there has been lately on the semantic and pragmatic properties of SMs, there are only a few works that have scrutinized what exactly these Ignorance Inferences amount to (see Schwarz 2013, 2016, Spychalska 2015, Mendia 2015). Moreover, most researchers have focused on the "numeral", that is, cases where SMs associate with a numeral, and so we know little about the "phrasal" case and the Ignorance Inferences it comes with. There seems to be a tacit assumption in the literatures that they behave the same. The present study shows that this is not the case. Ignorance Inferences that come with SMs are not the same across the board: they depend on the type of complement that the SM is associated with. In particular, it is shown that when the associate of the SM is totally ordered (e.g., when it is a numeral), the exhaustive interpretation of the prejacent must necessarily constitute an epistemic possibility for the speaker. However, when the associate of the SM constitutes a partial order, the exhaustive interpretation of the prejacent can but need to constitute an epistemic possibility for the speaker. These findings constitute are novel and provide further criteria to evaluate different competing theories of SMs.

INTRODUCTION. There are two well known and uncontroversial facts about Superlative Modifiers (SMs) like *at least* and *at most*: (i) that they can take a wide variety of elements as complements (Krifka 1999), and (ii) that they give rise to certain Ignorance Implicatures (IIs) (Nouwen 2010):

(1) a. Bill said: Ed has {at least / at most} four dogs ⇝ *ignorance about the exact number of dogs*
b. Bill said: {At least / At most} Liz and Sue came ⇝ *ignorance about exactly who came*

Both (1a) and (1b) may convey that Bill is ignorant about something: in (1a), about the exact number of dogs that Ed has, and in (1b) about who exactly came to the party. In recent work, Mendia (2015, *in prep*) argues that despite their apparent similarity, the IIs of (1a) and (1b) are in fact different. In particular, it is argued that (i) when SMs modify a scale which constitutes a Total Order (TO), the exhaustive interpretation of the prejacent *must* constitute an epistemic possibility for the speaker, but (ii) when SMs modify a scale which constitutes a Partial Order (PO), the exhaustive interpretation of the prejacent *can but need not* constitute an epistemic possibility for the speaker. This paper reports on an experimental investigation showing that this prediction is borne out. The results constitute a novel finding that has not been previously noted in the literature.

Motivation & Goals. Although there has been a number of recent studies investigating the properties of SMs, both from theoretical (Nouwen 2015, Kennedy 2015, and references therein) as well as experimental (Cummins and Katsos 2010, Coppock and Brochhagen 2013b, McNabb and Penka 2014, a.o.) perspectives, no study to my knowledge has scrutinized the properties of implicatures that come with SMs modifying Partially Ordered complements, as in (1b). Furthermore, the few extant theories suited to account for cases as in (1b) make divergent predictions about exactly what knowledge is compatible with the epistemic state of a speaker uttering an SM-statement. Thus, the goal of this study is to experimentally investigate the properties of IIs in sentences like (1b) and in turn evaluate the presently available accounts with respect to their ability to explain the results.

BACKGROUND. The experiment tests predictions made by three different accounts of IIs with SMs taking PO-complements.

***Theory I* (IS$^-$)**. The first proposal, developed by Coppock and Brochhagen (2013a) (C&B) is couched within Inquisitive Semantics. SMs are taken to require that there be minimally two epistemic possibilities that are compatible with the speaker's knowledge. The set of available possibilities is further constrained by the information state of the speaker, her epistemically accessible worldsif the speaker knows that Ed does not have 2 dogs she will factor out worlds where this possibility is alive. All that it is required of SMs is that they denote *any* two possibilities, but nothing is said about *which* possibilities.[1] As a consequence, (1a) with *at most* ($\mathsf{K_S}[\leq 4]$) will **not** entail that the speaker considers the prejacent to be an epistemic possibility ($\mathsf{P_S}[4]$). Suppose that Bill knows that Ed has either 2 or 3 dogs. C&B predict that (1a) with *at most* is felicitous, since there are two possibilities alive for the speaker –namely $\mathsf{P_S}[2]$ and $\mathsf{P_S}[3]$ (Spychalska 2015). For the same reasons, (1b) will **not** require that Bill mandatorily considers the possibility that just Liz and Sue came, $\mathsf{P_S}[\mathrm{L} \oplus \mathrm{S}]$. IIs are then derived by virtue SMs' signaling that the speaker is unable to restrict her epistemic states to a singleton.

***Theory II* (IS+)**. **IS+** is an amendment of **IS-**. In Coppock and Brochhagen (2013b), the authors propose a new pragmatic principle, the Maxim of Depictive Sincerity, whereby if a proposition $[\varphi]$ *highlights* a possibility p, then the speaker considers p possible, $\mathsf{P_S}[p]$. A possibility in $[\varphi]$ is highlighted if it is overtly expressed. Thus, since (1a) overtly expresses the possibility that Ed has 4 dogs, $[4]$ must be considered an epistemic possibility by the speaker, $\mathsf{P_S}[4]$. Extending Depictive Sincerity

1 Notational conventions: I enclose propositions in square brackets, such that $[\geq \varphi]$ stands for [*at least* φ], $[\leq \varphi]$ for [*at most* φ] and $[O\ \varphi]$ for [*only* φ]. I use $\mathsf{K_S}$ and $\mathsf{P_S}$ for the speaker oriented epistemic necessity and possibility operators.

to the conjunctive case, that Liz and Sue came must be considered an epistemic possibility in (1b), $\mathsf{P}_S[\mathrm{L} \oplus \mathrm{S}]$. Suppose again that Bill knows that Ed has 2 or 3 dogs. Contrary to **IS-**, **IS+** predicts that (1a) with *at most* is infelicitous, since the possibility that is highlighted (that Ed has 4 dogs) is not one of the two possibilities that are compatible with the speaker's knowledge. ***Theory III*** **(NG).** Working within a neo-Gricean framework, Mendia (2015) builds on earlier work by Schwarz (2013, *in press*), a.o., and defends the idea that IIs of SMs should be derived as Primary Implicatures. To do so, (i) SMs are assumed to form a Horn-Set with *only* (⟨*at least, only*⟩ and ⟨*at most, only*⟩), and (ii) the set of alternatives relevant for the Gricean calculus are provided by two independent mechanisms: (a) substitution of scale-mates within a Horn-Set (Sauerland 2004), and (b) substitution by replacing the focus-bearing constituent with contextually relevant alternatives (Rooth 1985). This account derives the IIs of SMs with TOs by creating a pair of symmetric alternatives. For (1a) with *at most*, $[\leq 4]$, we have the pair of Stronger Alternatives $[\leq 3]$ and $[O\ 4]$. The assertion, $\mathsf{K}_S[\leq 4]$, together with the Primary Implicatures $\neg\mathsf{K}_S[\leq 3]$ and $\neg\mathsf{K}_S[O\ 4]$ entail that each of the Stronger Alternatives must be an epistemic possibility, $\mathsf{P}_S[\leq 3] \wedge \mathsf{P}_S[O\ 4]$. (Negating any one of $\mathsf{P}_S[\leq 3]$ or $\mathsf{P}_S[O\ 4]$ would entail the truth of the other, contradicting the corresponding Primary Implicature.) However, the situation is different with POs. If (1b) with *at least* is uttered in a situation where Mary and Ann are also salient, no pair of symmetric alternatives can be derived. For $[\mathrm{L} \oplus \mathrm{S}]$ we have the Stronger Alternatives that $[O\ \mathrm{L} \oplus \mathrm{S}]$, $[\geq\ \mathrm{L} \oplus \mathrm{S} \oplus \mathrm{M}]$ and $[\geq\ \mathrm{L} \oplus \mathrm{S} \oplus \mathrm{A}]$, which, negated, provide the Primary Implicatures that $\neg\mathsf{K}_S[O\ \mathrm{L} \oplus \mathrm{S}]$, $\neg\mathsf{K}_S[\geq\ \mathrm{L} \oplus \mathrm{S} \oplus \mathrm{M}]$ and $\neg\mathsf{K}_S[\geq\ \mathrm{L} \oplus \mathrm{S} \oplus \mathrm{A}]$. Any one of the Stronger Alternatives can in fact be negated without fear of contradicting any Primary Implicature. Thus, under Mendia's (2015, *in prep.*) approach, the knowledge that is compatible with SMs *varies depending on the type of scale that SMs associate with*.

EXPERIMENT. The goal of the experiment is to test the predictions made by the three theories with respect to IIs. ***Method.*** 36 native English speakers were recruited via Amazon Mechanical Turk for an Acceptability Judgment Task. We crossed Type (TO *vs.* PO) and Condition (GOOD, BAD, TARGET) to create 30 critical items in a 2×3 factorial design. Quantifer (*at least* versus *at most*) was counterbalanced, with half of the items involving *at least*, and the other half *at most*. Items consisted of a short context and a short Q&A dialog. Subjects had to judge the naturalness of the answer to the question; after reading the dialog, they were asked the question *Is this response OK?*, which they had to answer by pressing a key for either 'Yes' or 'No'. Acceptability Judgments, as well as Response Times were collected. All TO items used numerals, whereas PO items involved plurals formed by conjunction (assuming a lattice-theoretic approach to plurals; Link 1983). ***Paradigm***:

Table 1: Predictions of the three theories

	Numeral (TO)			Conjunction (PO)		
IS-	$\mathsf{K}_S[SM\ \varphi]$	$\not\models$	$\mathsf{P}_S[\varphi]$	$\mathsf{K}_S[SM\ \varphi]$	$\not\models$	$\mathsf{P}_S[\varphi]$
IS+	$\mathsf{K}_S[SM\ \varphi]$	$\models$	$\mathsf{P}_S[\varphi]$	$\mathsf{K}_S[SM\ \varphi]$	$\models$	$\mathsf{P}_S[\varphi]$
NG	$\mathsf{K}_S[SM\ \varphi]$	$\models$	$\mathsf{P}_S[\varphi]$	$\mathsf{K}_S[SM\ \varphi]$	$\not\models$	$\mathsf{P}_S[\varphi]$

(2) *Context:* Sue is a teaching a class to four students: Mary, Liz, Al, and Bill. A colleague asks:

(3) **Type Total Order (Numeral)**
Question: How many students completed the quiz? *Answer*: I don't remember, at least two…
a. [GOOD]: maybe more. **b.** [BAD]: maybe one. **c.** [TARGET]: but not only two.

(4) **Type Partial Order (Conjunction)**
Question: Who completed the quiz? *Answer*: I don't remember, at least Mary and Liz…
a. [GOOD]: maybe somebody else. **b.** [BAD]: maybe only Liz. **c.** [TARGET]: but not only them.

The critical condition TARGET directly tests whether speakers accept utterances containing an SM, $[SM\ \varphi]$, together with the negation of its the prejacent, $\neg[\varphi]$, for this is the main point of divergence

among the three theories. ***Predictions.*** **IS**$^{-}$ predicts no difference between TARGET and GOOD, in either the TO Type (Numeral) nor in the PO Type (Conjunction), since for C&B (2013a) the prejacent of an utterance containing an SM is never required to be an epistemic possibility for the speaker. **IS**$^{+}$ predicts the opposite: TARGET should always behave like BAD with both TOs and POs, since –due to the Maxim of Depictive Sincerity– the prejacent is always required to be an epistemic possibility for the speaker. Finally, **NG** predicts that TARGET should behave like BAD only with TOs, since only with TOs can the pragmatic calculus generate a pair of symmetric alternatives (and this is required to derive the entailment that the prejacent is an epistemic possibility). In the case of POs no symmetric alternatives are generated, and so TARGET is predicted to be comparable to GOOD.

RESULTS & DISCUSSION. Figures 1 and 2 provide mean acceptance rates for both Types in all three conditions. The results show that speakers behave differently on TARGET depending on whether the scale modified by the SM is a TO or PO. When the scale is Total, TARGET acceptance rates are almost as low as for BAD (Fig. 1). However, when the scale is Partially Ordered, the rate of acceptance doubles, bringing the TARGET condition closer to the GOOD cases (Fig. 2). A mixed-effects logistic analysis confirmed these trends, revealing a significant Type*Condition interaction. Post-hoc comparisons showed that while acceptance rates did not vary significantly between Types (TO and PO) for GOOD and BAD conditions, the difference in acceptance for the TARGET conditions was statistically significant ($p < .001$). In addition, we also found a main effect of Quantifier, with *at most* items showing a lower rate of acceptance across the board ($p < .001$). The Type*Condition interaction was found to be much stronger with *at least* ($p < .001$). than with *at most*, although the latter also approach significane ($p < .1$). The results support the **NG** approach, which predicts the interaction found. However, notice that the acceptance of TARGET in the PO cases is not as low as BAD in Fig. 1, nor as high as GOOD in Fig. 2, as the **NG** account seems to predict. We suggest that calculating IIs of SMs in the TARGET require an additional step, which we call "consistency check": because the acceptability of an utterance of the form "$[SM\varphi] \wedge \neg[\varphi]$" varies depending on the structure of the scale involved, reasoning about the prejacent of SMs is costlier. In contrast, the judgments on GOOD/BAD conditions remains constant across scale Type, thus eliminating the need for "consistency checks". The RT data seems to support this hypothesis, as we observe latencies only with the TARGET conditions. On average, participants take longer in (i) accepting TARGET items in comparison to the corresponding GOOD items (5387ms vs. 4471ms) and (ii) rejecting TARGET items in comparison to the corresponding BAD cases (5433ms vs. 4355ms).

Figure 1: Type TO − Numeral

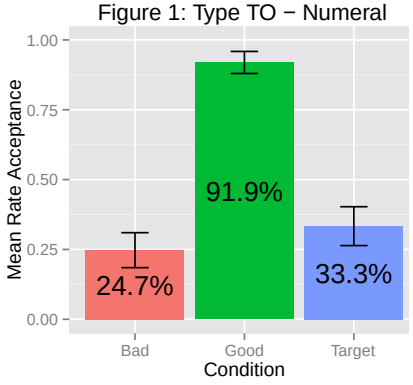
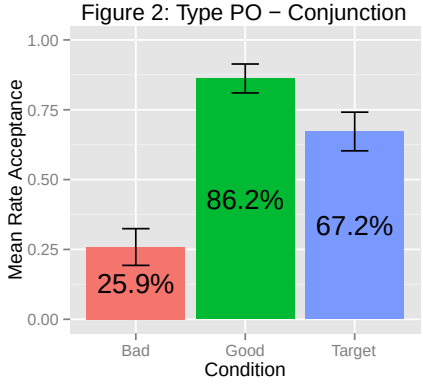


Figure 2: Type PO − Conjunction



CONCLUSION. The paper presented novel empirical evidence demonstrating that the structure of the scale that SMs modify matters for Implicature calculation. It was argued that an analysis along the lines of Mendia (2015) is most compatible with these results. However, we note that the experimental data paint a more complex picture than suggested in Mendia (2015). The full paper will elaborate on two main findings: (i) the greater processing costs associated with evaluating TARGET items, and (ii) the difference in overall acceptance rates between *at least* and *at most*.

References

Coppock, E. and Brochhagen, T. (2013a). Diagnosing truth, interactive sincerity, and depictive sincerity. In Snider, T. (ed.), *Proceedings of SALT* pp. 358–375.

Coppock, E. and Brochhagen, T. (2013b). Raising and resolving issues with scalar modifiers. *Semantics & Pragmatics* 6 pp. 1–57.

Cummins, C. and Katsos, N. (2010). Comparative and superlative quantifiers: pragmatic effects of comparison type. *Journal of Semantics* 27:3, pp. 271–305.

Kennedy, C. (2015). A "de-fregean" semantics (and neo-gricean pragmatics) for modified and unmodified numerals. *Semantics & Pragmatics* 8:10, pp. 1–44.

Krifka, M. (1999). At least some determiners aren't determiners. In Turner, K. (ed.), *The semantics/pragmatics interface from different points of view* vol. 1 of *Current research in the semantics/pragmatics interface*. Elsevier Science B.V.

Link, G. (1983). The logical analysis of plurals and mass terms. In Bauerle, R., Schwarze, C., and von Stechow, A. (eds.), *Meaning, use and interpretation of language* pp. 302–323. De Gruyter.

McNabb, Y. and Penka, D. (2014). The interpretation of superlative modifiers and deontic modals: an experimental investigation. In Etxeberria, U., Fălăuş, A. M., Irurtzun, A., and Leferman, B. (eds.), *Proceedings of Sinn und Bedeutung 18* pp. 271–288.

Mendia, J. A. (2015). Focusing on scales: Superlatives as scalar modifiers. Talk presented at NELS 46. Concordia University (Montreal).

Mendia, J. A. (in prep). Reasoning about uncertainty: superlative modifiers and primary implicatures. UMass Amherst.

Nouwen, R. (2010). Two kinds of modified numerals. *Semantics and pragmatics* 3:3, pp. 1–41.

Nouwen, R. (2015). Modified numerals: the epistemic effect. In Alonso-Ovalle, L. and Menéndez-Benito, P. (eds.), *Epistemic Indefinites*. Oxford University Press.

Rooth, M. (1985). *Association with Focus*. PhD thesis University of Massachusetts, Amherst.

Sauerland, U. (2004). Scalar implicatures in complex sentences. *Linguistics and Philosophy* 27:3, pp. 367–391.

Schwarz, B. (2013). At least and quantity implicature: Choices and consequences. In Maria Aloni, M. F. and Roelofsen, F. (eds.), *Proceedings of the 19th Amsterdam Colloquium* pp. 187–194.

Schwarz, B. (2016). Consistency preservation in quantity implicature: the case of *at least*. *Semantics & Pragmatics*.

Schwarz, B. (in press). Consistency preservation in quantity implicature: the case of *at least*. To appear in Semantics & Pragmatics.

Spychalska, M. (2015). At least not false, at most possible: between truth and assertibility of superlative quantifiers. *Synthese* pp. 1–32.